Suggested Solutions to: Regular Exam, Spring 2019 Industrial Organization June 3, 2019

This version: June 24, 2019

Question 1: Leasing a durable good

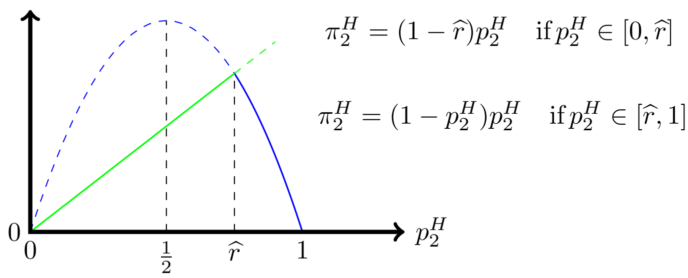
(a) Solve for the equilibrium value of $\widehat{r}$ and show that the solution you have found is indeed an equilibrium. You may assume that the second-order conditions are satisfied.

As stated in the question, we look for an equilibrium that is characterized by a cutoff value $\widehat{r}$, such that a consumer leases in the first period if and only if $r \geq \widehat{r}$. We can solve for such an equilibrium by considering all the stages of the model where an economic agent (the firm or the consumers) makes a choice, and ensure that these choices are made optimally (given that the agent correctly anticipates decisions made later in the game).

- **Stage 4 and 3 (the "L-market")**: Consumers with $r \in [0, \widehat{r}]$ give rise to the following demand schedule: $\boxed{q_2^L = \widehat{r} - p_2^L.}$
  - Period 2 profits, $\pi_2^L = \left(\widehat{r} - p_2^L\right)\left(p_2^L - c\right)$, maximized at $\boxed{p_2^L = \frac{\widehat{r}+c}{2}.}$
- **Stage 4 and 3 (the "H-market")**: Consumers with $r \in (\widehat{r}, 1]$ give rise to the following demand schedule:

$$\boxed{q_2^H = \begin{cases} 1 - p_2^H & \text{if } p_2^H \in [\widehat{r}, 1] \\ 1 - \widehat{r} & \text{if } p_2^H \in [0, \widehat{r}]. \end{cases}}$$

  - Given that $\widehat{r} \geq \frac{1}{2}$, the price that maximizes period 2 profits in the H-market, $\pi_2^H = q_2^H p_2^H$, is $\boxed{p_2^H = \widehat{r}}$; cf. the figure below.[1]



- **Stage 2**: Given the period 1 price $\boxed{p_1}$ and the period 2 prices $\boxed{p_2^L = \frac{\widehat{r}+c}{2}}$ and $\boxed{p_2^H = \frac{1}{2}}$, a consumer with $r = \widehat{r}$ is indifferent between buying in period 1 and not doing that:

$$r - p_1 + \delta\left[r - \overbrace{p_2^H}^{=\widehat{r}}\right] = 0 + \delta\left[r - \overbrace{p_2^L}^{=\frac{\widehat{r}+c}{2}}\right]$$

  So $\widehat{r}$ must satisfy

$$\widehat{r} - p_1 = \delta\left(\widehat{r} - \frac{\widehat{r}+c}{2}\right) \Leftrightarrow \boxed{\widehat{r} = \frac{2p_1 - \delta c}{2-\delta}.}$$

- **Stage 1**: The firm's period 1 objective, using $\boxed{p_2^L = \frac{\widehat{r}+c}{2},\ p_2^H = \widehat{r}}$:

$$\pi_1 + \beta\pi_2 =$$

$$\overbrace{(1-\widehat{r})}^{=q_1}(p_1 - c) + \beta\overbrace{\left(\widehat{r} - p_2^L\right)}^{=q_2^L}\left(p_2^L - c\right) + \beta\overbrace{\left(1 - p_2^H\right)}^{=q_2^H}p_2^H =$$

$$(1-\widehat{r})(p_1 - c) + \beta\left[\left(\frac{\widehat{r}-c}{2}\right)^2 + (1-\widehat{r})\widehat{r}\right] =$$

$$(1-\widehat{r})(\widehat{r}-c)\frac{2-\delta}{2} + \frac{\beta}{4}\left[(\widehat{r}-c)^2 + 4(1-\widehat{r})\widehat{r}\right].$$

  - The last equality uses the relationship $p_1 - c = \frac{2-\delta}{2}(\widehat{r} - c)$. To simplify the alge-

[1] According to the question, the firm does not incur any new production costs in this situation, as these goods are already in existence.

bra, let the firm choose $\widehat{r}$ instead of p_1—given the one-to-one relationship between $\widehat{r}$ instead of p_1, this is equivalent.

- FOC: $\partial\left(\pi_1+\beta\pi_2\right)/\partial\widehat{r}=$

$$
\begin{aligned}
& \left(1+c-2\widehat{r}\right)\frac{2-\delta}{2}+\frac{\delta}{2}\left[\left(\widehat{r}-c\right)+2\left(1-2\widehat{r}\right)\right] \\
= \quad & 0 \Leftrightarrow \boxed{\widehat{r}^{*}=\frac{2(1+c)+\delta(1-2c)}{4+\delta}}. \qquad (1)
\end{aligned}
$$

- Our analysis is valid only for $\widehat{r}\in[\frac{1}{2},1)$. By working through some algebra, one can verify that we have both $\widehat{r}\geq\frac{1}{2}$ and $\widehat{r}<1$. One can also see from the algebra above that the prices take values that are fully feasible. Thus, we can conclude that (1) indeed is an equilibrium value of $\widehat{r}$.

(b) Denote total surplus (i.e., the sum of firm profit and consumer surplus) for the market in period t by W_t, for $t=1,2$. Write up expressions for W_1 and W_2, as functions of $\widehat{r}$, p_1, p_2^L, and p_2^H (i.e., do *not* plug in the equilibrium values of this cutoff value and these prices).

- You are encouraged to attempt this question also if you have failed to answer part (a).

In period 1, consumers with $r\in[\widehat{r},1]$ consume the good, and the per-unit production cost is c. Therefore, total surplus in period 1 is given by

$$W_1=\int_{\widehat{r}}^{1}(r-c)dr.$$

Alternatively, total surplus in period 1 can be written as the sum of first-period profits and consumer surplus or $W_1=\Pi_1+CS_1$, where $\Pi_1=(1-\widehat{r})(p_1-c)$ and

$$CS_1=\int_{\widehat{r}}^{1}(r-p_1)dr.$$

In period 2, consumers with $r\in[p_2^L,1]$ consume the good. Moreover, the per-unit production cost for those with $r\in[p_2^L,\widehat{r}]$ is c, and the per-unit production cost for those with $r\in[\widehat{r},1]$ is zero. Therefore, total surplus in period 2 is given by

$$W_2=\int_{p_2^L}^{\widehat{r}}(r-c)dr+\int_{\widehat{r}}^{1}rdr.$$

Alternatively, total surplus in period 2 can be written as the sum of second-period profits and consumer surplus or $W_2=\Pi_2+CS_2$, where $\Pi_2=(p_2^L-c)(\widehat{r}-p_2^L)+p_2^H(1-\widehat{r})$ and

$$CS_2=\int_{p_2^L}^{\widehat{r}}(r-p_2^L)dr+\int_{\widehat{r}}^{1}(r-p_2^H)dr.$$

Question 2: Strategic delegation

To the external examiner: This question is identical to a question in a problem set that the students discussed in an exercise class.

Part (a)

The game consists of two stages. At the first stage the owners, independently and simultaneously, choose an instruction P_i or R_i. At the second stage we have four different possibilities, depending on what instructions the owners have chosen: both firms are profit maximizers, (P_1,P_2); both firms are revenue maximizers, (R_1,R_2); or one is a profit maximizer and the other is a revenue maximizer, (P_1,R_2) or (R_1,P_2). Given these objectives, the managers choose, independently and simultaneously, a quantity q_i.

- We can solve for the subgame-perfect Nash equilibria of the model by backward induction. We therefore start by solving the four second-stage subgames.

- **The case (P_1,P_2).** Each firm maximizes

$$
\begin{aligned}
& \left[45-9\left(q_1+q_2\right)\right]q_i-9q_i \\
= \quad & \left[36-9\left(q_1+q_2\right)\right]q_i.
\end{aligned}
$$

The FOCs for the two firms are

$$-9q_1+\left[36-9\left(q_1+q_2\right)\right]=0$$

and

$$-9q_2+\left[36-9\left(q_1+q_2\right)\right]=0.$$

Solving these equations for q_1 and q_2 yields

$$\left(q_1^{PP},q_2^{PP}\right)=\left(\frac{4}{3},\frac{4}{3}\right).$$

The profit levels given these outputs are

$$\pi_1^{PP}=\left[45-9\left(q_1^{PP}+q_2^{PP}\right)\right]q_1^{PP}-9q_1^{PP}=16$$

and

$$\pi_2^{PP}=\left[45-9\left(q_1^{PP}+q_2^{PP}\right)\right]q_2^{PP}-9q_2^{PP}=16.$$

- **The case** (R_1, R_2)**.** Each firm maximizes its revenues
$$[45 - 9(q_1 + q_2)]\, q_i.$$
The FOCs for the two firms are
$$-9q_1 + [45 - 9(q_1 + q_2)] = 0$$
and
$$-9q_2 + [45 - 9(q_1 + q_2)] = 0.$$
Solving these equations for q_1 and q_2 yields
$$\left(q_1^{RR}, q_2^{RR}\right) = \left(\frac{5}{3}, \frac{5}{3}\right).$$
The profit levels given these outputs are
$$\pi_1^{RR} = \left[45 - 9\left(q_1^{RR} + q_2^{RR}\right)\right] q_1^{RR} - 9q_1^{RR} = 10$$
and
$$\pi_2^{RR} = \left[45 - 9\left(q_1^{RR} + q_2^{RR}\right)\right] q_2^{RR} - 9q_2^{RR} = 10.$$

- **The case** (P_1, R_2)**.** Firm 1 maximizes its profit
$$\begin{aligned} & [45 - 9(q_1 + q_2)]\, q_i - 9q_i \\ = \; & [36 - 9(q_1 + q_2)]\, q_i. \end{aligned}$$
Firm 1's FOC is
$$-9q_1 + [36 - 9(q_1 + q_2)] = 0. \qquad (2)$$
Firm 2 maximizes its revenues
$$[45 - 9(q_1 + q_2)]\, q_i.$$
Firm 2's FOC is
$$-9q_2 + [45 - 9(q_1 + q_2)] = 0. \qquad (3)$$
Solving equations (2) and (3) for q_1 and q_2 yields
$$\left(q_1^{PR}, q_2^{PR}\right) = (1, 2).$$
The profit levels given these outputs are
$$\pi_1^{PR} = \left[45 - 9\left(q_1^{PR} + q_2^{PR}\right)\right] q_1^{PR} - 9q_1^{PR} = 9$$
and
$$\pi_2^{PR} = \left[45 - 9\left(q_1^{PR} + q_2^{PR}\right)\right] q_2^{PR} - 9q_2^{PR} = 18.$$

- **The case** (R_1, P_2)**.** This is symmetric to the case (P_1, R_2). Therefore, $\left(q_1^{RP}, q_2^{RP}\right) = (2, 1)$,
$$\pi_1^{RP} = 18,$$
and
$$\pi_2^{RP} = 9.$$

- We have now solved all the stage 2 subgames and derived expressions for the equilibrium profit levels in all of these. Using these profit levels we can illustrate the stage 1 interaction between O_1 and O_2 in a game matrix (where O_1 is the row player and O_2 is the column player):

	P_2	R_2
P_1	16, 16	9, 18
R_1	18, 9	10, 10

We see that each player has a strictly dominant strategy and that, in particular, the unique Nash equilibrium of the stage 1 game is that both owners choose revenue maximization, (R_1, R_2).

- **Conclusion:** the game has a unique SPNE. In this equilibrium, both owners choose revenue maximization, (R_1, R_2). In the stage 2 equilibrium path subgame, the managers choose $\left(q_1^{RR}, q_2^{RR}\right) = \left(\frac{5}{3}, \frac{5}{3}\right)$. In the three off-the-equilibrium path subgames, the managers choose $\left(q_1^{PP}, q_2^{PP}\right) = \left(\frac{4}{3}, \frac{4}{3}\right)$, $\left(q_1^{PR}, q_2^{PR}\right) = (1, 2)$, and $\left(q_1^{RP}, q_2^{RP}\right) = (2, 1)$.

Part (b)

Interpretation: The owners would be better off if they both chose to instruct their manager to maximize profit. The reason why this cannot be part of an equilibrium is that each firm can gain by unilaterally instruct its own manager to maximize revenues instead. Why is this the case? First, a manager who maximizes revenues will be more aggressive (i.e., produce more) than a profit maximizing manager. Second, the rival manager, expecting this behavior, will respond by producing less (since the firms' outputs are strategic substitutes). This will increase the first firm's market share and profit.

- If the managers' choice variables had been strategic complements instead we should expect the opposite result: each firm would like to make the rival behave in a way that is good for the own profits (i.e., charge a high price or choose a small quantity). If the choice variables are strategic complements, this means that to induce the rival to behave like that a firm should behave in the same way itself (i.e., charge a high price or choose a small quantity). Therefore, an owner could gain by instructing its manager to be relatively non-aggressive (i.e., to have a strong incentive to charge a high price or choose a small quantity) — this can be achieved by instructing the manager to maximize profits rather than revenues.

- The assumption that the instruction is observable by the rival firm is crucial. Without that assumption, an owner would always want the own manager to maximize profits (but maybe still be *telling* the rival manager that the instruction was R). The point with choosing R is that then the rival *knows* this (and knows that this choice is irreversible), which will (in the model with strategic substitutes) have a beneficial effect on the rival manager's optimal choice at the second stage.